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Some Exact Bianchi Type V Perfect Fluid Solutions with Heat Flow

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Abstract The variation law for generalized mean Hubble's parameter is discussed in a spatially homogeneous and anisotropic Bianchi type V space-time with perfect fluid along with heat-conduction. The variation law for Hubble's parameter, that yields a constant value of deceleration parameter, generates two types of solutions for the average scale factor, one is of power-law type and other one of exponential form. Using these two forms of the average scale factor, exact solutions of Einstein field equations with a perfect fluid and heat conduction are presented for a Bianchi type V space-time, which represent expanding singular and non-singular cosmological models. We find that the constant value of deceleration parameter is reasonable for the present day universe. The physical and geometrical properties of the models are also discussed in detail.

Keywords Cosmology · Hubble's parameter · Deceleration parameter · Bianchi models · Perfect fluid · Heat conduction

1 Introduction

The purpose of this work is to study Bianchi type V spatially homogeneous and anisotropic cosmologies where the source of the gravitational field is a perfect fluid with heat flow. It is certainly of interest to study cosmologies with a richer structure, both geometrically and physically, than the standard perfect fluid Friedmann-Robertson-Walker (FRW) models. Bianchi type V models are of particular interest since they are sufficiently complex as the

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Einstein tensor has off-diagonal terms while at the same time are a simple generalization of the negative curvature FRW models. The cosmological models of this type have a significant role in the description of the universe in the early stages of its evolution. The matter content of the universe is satisfactorily described by a perfect fluid. As the matter is not expected to attain thermal equilibrium in the early stages of the evolution of the universe, it is evident that there would be heat flow in the universe.

The effect of heat flow in the evolution of the universe has been investigated by several authors such as Deng [1], Mukherjee [2], Novello and Reboucas [3], Ray [4], Reboucas and de Limma [5], Reboucas [6], Bradley and Sviestins [7] and Sviestins [8]. Banerjee and Sanyal [9] considered Bianchi type V cosmologies with viscosity and heat flow. It has also been shown that it is possible for dissipative Bianchi type V universe models not to be in thermal equilibrium in their early stages. Coley [10] investigated Bianchi type V spatially homogeneous imperfect fluid cosmological models which contain both viscosity and heat flow. Coley and Hoogen [11] also generalized the work of Coley and Dunn [12] who assumed a locally rotationally symmetric Bianchi type V metric for an imperfect fluid source with both viscosity and heat conduction. Recently Singh [13] presented some new Bianchi type V cosmological models in the presence of perfect fluid with heat flow.

The Einstein's field equations are a coupled system of highly non-linear differential equations and we seek physical solutions to the field equations for their applications in cosmology and astrophysics by making certain assumptions either at the cost of the physics or simply for mathematical convenience. Many authors have obtained solutions to the field equations by different approaches. Solutions to the field equations may be generated by applying a law of variation for Hubble's parameter, which was initially proposed by Berman [14]. The variation of Hubble's parameter is not inconsistent with observation and has the advantage of providing simple functional forms of the average scale factor. This variation-law also yields a constant deceleration parameter. The law of variation for Hubble's parameter gives a new approach for solving field equations, an approach that is quite general and suitable for the description of present day universe. The cosmological models with constant deceleration parameter have been studied by Berman [14], Berman and Gomide [15]. Johri and Desikan [16], Singh and Desikan [17], Maharaj and Naidoo [18], Pradhan et al. [19] and others.

In Sect. 2 of this work, we present the model of the universe and the expressions for basic equations. We discuss, in Sect. 3, the law of variation of the Hubble's parameter and derive two types of solutions for the average scale factor of the model, one is of power-law type and other of exponential form for Bianchi type V metric. Using these two forms of the average scale factor, two classes of exact solutions of Einstein field equations in the presence of a perfect fluid with heat flow, which correspond to singular and non-singular cosmological models, are presented. We also discuss the physical and geometrical behaviors of the models in each cosmology. Some concluding remarks are given in Sect. 4.

2 Derivation of Basic Equations

As a gravitational field we consider the Bianchi type V cosmological space-time given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}\left[B^2(t)dy^2 + C^2(t)dz^2\right], \tag{1}$$

where A, B, C are functions of cosmic time t and m is a constant. The average scale factor a of the metric (1) is defined as

$$a = (ABC)^{1/3}, \tag{2}$$

where volume scale factor V is given by

$$V = a^3. \tag{3}$$

We define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \tag{4}$$

where $H_1 = \dot{A}/A$, $H_2 = \dot{B}/B$ and $H_3 = \dot{C}/C$ are the directional Hubble's parameters in the directions of x, y and z respectively. A dot denotes differentiation with respect to t. With the help of the scale factors, one can define the following variables.

From (2)–(4), we obtain

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \tag{5}$$

An important observational quantity is the deceleration parameter q defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{6}$$

The physical quantities of observational interest in cosmology are the expansion scalar θ, the shear scalar σ and the average anisotropy parameter $Am(\geq 0)$ which are defined as

$$\theta = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{7}$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{6}, \tag{8}$$

$$Am = \frac{1}{3}\sum_{\mu=1}^{3}\left(\frac{\Delta H_\mu}{H}\right)^2, \tag{9}$$

where $\Delta H_\mu = H_\mu - H$ $(\mu = 1, 2, 3)$.

3 Field Equations and Their Solutions

The influence of the perfect fluid with heat flow in the evolution of the universe is performed by means of its energy-momentum tensor, which acts as the source of the corresponding gravitational field. The energy-momentum tensor of a perfect fluid with heat conduction has the form [10–13]:

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p g_{\mu\nu} + h_\mu u_\nu + h_\nu u_\mu, \tag{10}$$

where p is the thermodynamic pressure, ρ the energy density, u_μ the four-velocity of the fluid and h_μ is the heat flow vector satisfying

$$h^\mu u_\mu = 0. \tag{11}$$

Let us assume that, in the Bianchi type model under consideration, the fluid four-velocity is comoving. Then we take $u_\mu = (0, 0, 0, 1)$.

The Einstein field equations in a system of units $8\pi G = c = 1$ without cosmological constant can be written as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -T_{\mu\nu}. \tag{12}$$

The field equations (12) and (11) imply that the heat flow is in x-direction only i.e. $h^{\mu} = (h_1(t), 0, 0, 0)$. In a co-moving coordinate system, the explicit forms of the field equations (12), in view of (1) and (10), can be written as

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3m^2}{A^2} = \rho, \tag{13}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{m^2}{A^2} = -p, \tag{14}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -p, \tag{15}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -p, \tag{16}$$

$$m\left(2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = h_1. \tag{17}$$

The law of energy-conservation equation $T^{\mu\nu}_{;\nu} = 0$ gives

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{2m}{A^2} h_1. \tag{18}$$

We also recall that (13)–(17) can be written in terms of H, σ^2 and q as

$$\rho = 3H^2 - \sigma^2 - \frac{3m^2}{A^2}, \tag{19}$$

$$p = H^2(2q - 1) - \sigma^2 + \frac{m^2}{A^2}. \tag{20}$$

Let us now solve Einstein field equations (13)–(17). Here we follow the approach of Saha and Rikhvitsky [20] and Singh and Chaubey [21] to solve the field equations. Subtracting (16) from (15), (16) from (14) and (15) from (14), we get the following three relations, respectively

$$\frac{A}{B} = d_1 \exp\left(k_1 \int \frac{dt}{a^3}\right). \tag{21}$$

Analogically, we find the other relations

$$\frac{A}{C} = d_2 \exp\left(k_2 \int \frac{dt}{a^3}\right), \tag{22}$$

$$\frac{B}{C} = d_3 \exp\left(k_3 \int \frac{dt}{a^3}\right), \tag{23}$$

where d_1, d_2, d_3 and k_1, k_2, k_3 are constants of integration. From (21)–(23), the metric functions can be written explicitly as

$$A(t) = l_1 a.\exp\left(\frac{X_1}{3}\int \frac{dt}{a^3}\right), \tag{24}$$

$$B(t) = l_2 a.\exp\left(\frac{X_2}{3}\int \frac{dt}{a^3}\right), \tag{25}$$

$$C(t) = l_3 a.\exp\left(\frac{X_3}{3}\int \frac{dt}{a^3}\right), \tag{26}$$

where

$$l_1 = \sqrt[3]{d_1 d_2}, \qquad l_2 = \sqrt[3]{d_1{}^{-1} d_3}, \qquad l_3 = \sqrt[3]{(d_2 d_3)^{-1}},$$
$$X_1 = k_1 + k_2, \qquad X_2 = k_3 - k_1, \qquad X_3 = -(k_2 + k_3).$$

The constants X_1, X_2, X_3 and l_1, l_2, l_3 satisfy the relations

$$X_1 + X_2 + X_3 = 0 \quad \text{and} \quad l_1 l_2 l_3 = 1. \tag{27}$$

Thus the metric functions are found explicitly in terms of the average scale factor a.

In order to find the solution of the metric functions in terms of cosmic time t, let us consider that the generalized mean Hubble parameter H is related to the average scale factor a by the relation

$$H = l a^{-n}, \tag{28}$$

where $l > 0$ and $n (\geq 0)$ are constants. Such type of relation has already been considered by Berman [14], and Berman and Gomide [15] for solving FRW models. Such relation gives a constant value of deceleration parameter. It may be noted that though the current observations of SNe Ia and CMB favour accelerating models ($q < 0$), but they do not altogether rule out the decelerating ones which are also consistent with these observations. Later on, several authors [see [22] and references therein] have considered FRW cosmological models with constant deceleration parameter. Singh and Kumar [22–24] and Kumar and Singh [25] extended this work to anisotropic Bianchi types I and II cosmological models in general relativity and some scalar-tensor theories with constant deceleration parameter. In this paper, our intention is to solve the field equations of Bianchi V perfect fluid with heat flow by using relation (28), since the relation (28) refers to Bianchi V space-time in any physical context.

From (5) and (28), we find that

$$\dot{a} = l a^{-n+1}, \tag{29}$$

$$\ddot{a} = -l^2 (n-1) a^{-2n+1}. \tag{30}$$

Using (29) and (30) in (6), we obtain

$$q = n - 1. \tag{31}$$

We see that q is constant. The sign of q indicates whether the model inflates or not. The positive sign of q corresponds to standard decelerating model whereas the negative sign indicates inflation.

From (29), we obtain the laws of variation of the average scale factor of the forms

$$a = (nlt + c_1)^{1/n} \tag{32}$$

for $n \neq 0$ and

$$a = c_2 \exp(lt) \tag{33}$$

for $n = 0$, where c_1 and c_2 are constants of integration.

Now we solve the exact solutions of quadrature equations (24)–(26) and corresponding energy density and pressure from (19) and (20) in the following two subsections depending on the values of n as defined in (32) and (33).

3.1 Solution with $n \neq 0$

Using the power-law form of the average scale factor $a(t)$, as given by (32), into (24)–(26), the solutions for the metric functions can be written as

$$A(t) = l_1 (nlt + c_1)^{1/n} \exp\left[\frac{X_1}{3l(n-3)} (nlt + c_1)^{(n-3)/n}\right], \tag{34}$$

$$B(t) = l_2 (nlt + c_1)^{1/n} \exp\left[\frac{X_2}{3l(n-3)} (nlt + c_1)^{(n-3)/n}\right], \tag{35}$$

$$C(t) = l_3 (nlt + c_1)^{1/n} \exp\left[\frac{X_3}{3l(n-3)} (nlt + c_1)^{(n-3)/n}\right], \tag{36}$$

where $n \neq 3$. Substituting (34)–(36) in (17), we find that

$$h_1 = \frac{mX_1}{(nlt + c_1)^{3/n}}. \tag{37}$$

The energy density and pressure as calculated from (19) and (20) have the values given by,

$$\begin{aligned} \rho = {} & 3l^2 (nlt + c_1)^{-2} - \frac{({X_1}^2 + {X_2}^2 + {X_3}^2)}{18} (nlt + c_1)^{-6/n} \\ & - \frac{3m^2}{{l_1}^2} (nlt + c_1)^{-2/n} \exp\left[\frac{-2X_1}{3l(n-3)} (nlt + c_1)^{(n-3)/n}\right], \end{aligned} \tag{38}$$

$$\begin{aligned} p = {} & l^2(2n-3) (nlt + c_1)^{-2} - \frac{({X_1}^2 + {X_2}^2 + {X_3}^2)}{18} (nlt + c_1)^{-6/n} \\ & + \frac{m^2}{{l_1}^2} (nlt + c_1)^{-2/n} \exp\left[\frac{-2X_1}{3l(n-3)} (nlt + c_1)^{(n-3)/n}\right]. \end{aligned} \tag{39}$$

Using above solutions (34)–(39), it can easily be seen that the energy conservation equation (18) is identically satisfied. Therefore, we have obtained exact solutions of Bianchi type-V cosmological model with perfect fluid and heat flow.

The directional Hubble's parameters H_1, H_2 and H_3 are given by

$$H_1 = \frac{l}{(nlt + c_1)} + \frac{X_1}{3\,(nlt + c_1)^{3/n}}, \tag{40}$$

$$H_2 = \frac{l}{(nlt + c_1)} + \frac{X_2}{3\,(nlt + c_1)^{3/n}}, \tag{41}$$

$$H_3 = \frac{l}{(nlt + c_1)} + \frac{X_3}{3\,(nlt + c_1)^{3/n}}, \tag{42}$$

whereas the mean Hubble's parameter is given by

$$H = l\,(nlt + c_1)^{-1}. \tag{43}$$

The kinematical parameter θ, σ^2, Am and q have been obtained as

$$\theta = 3l(nlt + c_1)^{-1}, \tag{44}$$

$$\sigma^2 = \frac{({X_1}^2 + {X_2}^2 + {X_3}^2)}{18}(nlt + c_1)^{-\frac{6}{n}}, \tag{45}$$

$$Am = \frac{({X_1}^2 + {X_2}^2 + {X_3}^2)}{27l^2}(nlt + c_1)^{\frac{2(n-3)}{n}}, \tag{46}$$

$$q = n - 1. \tag{47}$$

At the initial moment $t = t_1$ where $t_1 = -c_1/nl$, the physical parameters ρ, p and h_1 tend to infinity. Therefore, the universe starts evolving from initial singularity $t = t_1$, with infinite density, infinite internal pressure and infinite internal heat flow. At the initial stage of expansion ρ, p and Hubble parameter are large and θ decreases with the power-law type of expansion of the universe. The three scale factors are monotonically increasing function of time. The deceleration parameter is constant. We observe that the spatial scale factors are zero at the initial moment $t = t_1$. The model has point singularity. The heat conduction is decreasing function of time and is maximum at the initial epoch. The scale factors tend to infinity whereas ρ and p tend to zero as $t \to \infty$. The dynamics of the mean anisotropy parameter depends on the value of n. For $n < 3$, Am has singular state, with infinite energy density and zero scale factors. For small time, Am is increasing and in the large time limits, it ends in a homogeneous and isotropic state. The model approaches isotropic during the late time of its evolution. The heat conduction diminishes as $t \to 8$. The anisotropy parameter Am will end in a homogeneous and isotropic state for large time. All physical and kinematical parameters tend to zero as t tends to infinity. We also find that $\lim_{t\to\infty} \sigma^2/\theta = 0$, which means that this model of the universe approaches isotropy during the late time of its evolution.

3.2 Solution with $n = 0$

In this case we present an exponentially expanding non-singular cosmological model. Using (33) into the quadratures (24)–(26), we obtain the solutions

$$A(t) = c_2 l_1 \exp\left[lt - \frac{X_1}{9l{c_2}^3}\exp(-3lt)\right], \tag{48}$$

$$B(t) = c_2 l_2 \exp\left[lt - \frac{X_2}{9l{c_2}^3}\exp(-3lt)\right], \tag{49}$$

$$C(t) = c_2 l_3 \exp\left[lt - \frac{X_3}{9l{c_2}^3}\exp(-3lt)\right], \tag{50}$$

$$h_1 = \frac{mX_1}{{c_2}^3}\exp(-3lt). \tag{51}$$

The energy-density and pressure have expressions

$$\rho = 3l^2 - \frac{({X_1}^2 + {X_2}^2 + {X_3}^2)}{18{c_2}^6}\exp(-6lt) - \frac{3m^2}{{c_2}^2{l_1}^2}\exp\left[-2\left(lt - \frac{X_1}{9l{c_2}^3}\exp(-3lt)\right)\right], \quad (52)$$

$$p = -3l^2 - \frac{({X_1}^2 + {X_2}^2 + {X_3}^2)}{18{c_2}^6}\exp(-6Dt) + \frac{m^2}{{c_2}^2{l_1}^2}\exp\left[-2\left(lt - \frac{X_1}{9l{c_2}^3}\exp(-3lt)\right)\right]. \quad (53)$$

Substituting (48)–(53), it can easily be seen that the conservation equation (18) is identically satisfied.

The directional Hubble's parameters H_1, H_2 and H_3 are given by

$$H_1 = l + \frac{X_1}{3{c_2}^3}\exp(-3lt), \quad (54)$$

$$H_2 = l + \frac{X_2}{3{c_2}^3}\exp(-3lt), \quad (55)$$

$$H_3 = l + \frac{X_3}{3{c_2}^3}\exp(-3lt), \quad (56)$$

whereas the average scale factor is $H = l$. The kinematical parameters θ, σ^2, Am and q are

$$\theta = 3l, \quad (57)$$

$$\sigma^2 = \frac{({X_1}^2 + {X_2}^2 + {X_3}^2)}{18{c_2}^6}\exp(-6lt), \quad (58)$$

$$Am = \frac{({X_1}^2 + {X_2}^2 + {X_3}^2)}{27l^2{c_2}^6}\exp(-6lt), \quad (59)$$

$$q = -1. \quad (60)$$

The negative value of q indicates inflation. We observe that the physical and kinematical quantities are all constants at $t = 0$. The heat conduction is decreasing function of time and is constant at $t = 0$. The heat flow diminishes for large time. This shows that the universe starts evolving with constant volume and expands exponentially. We find that the directional Hubble's parameters are time-dependent while the average Hubble parameter is constant. The expansion scalar is constant throughout the time of evolution. The pressure assumes a constant negative value as $t \to \infty$, which means that the universe is accelerating in the later stage of its evolution. The evolution of the universe in such a scenario is in consistent with the present day observations predicting an accelerated expansion. The heat function dies out for large time. The kinematical parameters tend to zero as $t \to \infty$. Also we find that $\frac{\sigma^2}{\theta}$ tends to zero as $t \to \infty$, which shows that this inflationary universe eventually becomes isotropic for large t. This cosmological model is non-singular.

4 Conclusion

We have presented a method for obtaining rotationless, anisotropic exact solutions of Einstein's equations for Bianchi type V space-time with constant deceleration parameter and perfect fluid along with heat conduction as matter sources. We have discussed the variation-law for Hubble's parameter in a spatially homogeneous and anisotropic Bianchi type V space-time and have derived two laws of variation of the average scale factor, one is of power-law type and the other of exponential form, which yield the positive and negative values of the deceleration parameter, respectively. Exact solutions of Einstein field equations with perfect fluid and heat conduction have been obtained for Bianchi type V space-time in these two types of cosmologies. In the cosmology with the power-law variation of the average scale factor, the solution corresponds to a cosmological model which starts expanding from the singular state with positive deceleration parameter. The heat conduction is decreasing function of time and it was maximum at early stages of the evolution of universe. In the case of exponential variation of the average scale factor we have presented an accelerating non-singular model of the universe. We have also discussed the physical and kinematical behaviors of the models in two types of cosmologies. The models presented in this paper could give an appropriate description of the evolution of universe. More realistic models may be analyzed by using this technique, which may lead to interesting and different physical behaviors of the evolution of universe.

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